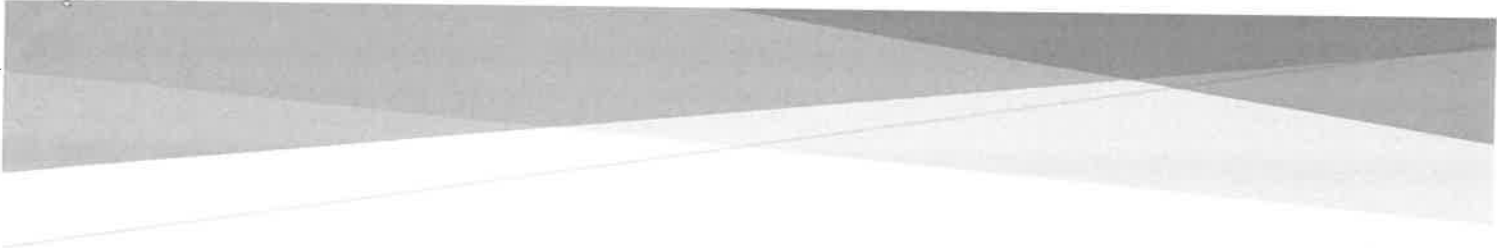


STUDENT FEEDBACK FORM-SHAUN FARTHING

This section details the *extent* to which the assessment criteria are demonstrated by you, which in turn determines your mark. The marks available for each category of skill are shown. Lecturers will use the space provided to comment on the achievement of the task(s), including those areas in which you have performed well and areas that would benefit from development/improvement.

Generic Assessment Criteria	Marks available	Marks awarded
1. Engagement with Literature Skills		
Your work has shown that you engaged with an extensive range of relevant and credible literature. Consistently accurate application of referencing. The student has clearly shown that he has all-important skills needed to define the essential ideas from sufficient information.	15	12
2. Knowledge and Understanding Skills		
The report shows good effort and good understanding of the topics covered. Good structure, detail and data presented in clear and appropriate format throughout.	20	20
3. Cognitive and Intellectual Skills		
Your work showed that you can summarise scientific facts into your own words without compromising academic style. There is a coherent flow in the writing and the reader can easily follow the simulations process and the analysis of the results. You have made it interesting to read and have also made attempts at drawing conclusions for the area you studied. Very good critical evaluation and analysis of the concept.	20	14
4. Practical Application Skills		
A good and appropriate application of established methods and techniques. The report clearly demonstrates the student has a clear understanding of studies to date and also shows the student's ability to work as an individual. Good detail provided throughout the report with all aspects being suitably and sufficiently carried out. All tables and figures presented are thoroughly discussed in the assignment. All aspects have been covered in excellent detail.	20	12
5. Transferable Skills for Life and Professional Practice		
Your work is coherent, fluent, well-structured and organised. Your work also shows that you can work very well autonomously. Your work demonstrates very good skills for employment requiring the exercise of personal responsibility and decision-making, with just occasional minor weakness.	25	16

Assessment Mark (Assessment marks are subject to ratification at the Exam Board. These comments and marks are to give feedback on module work and are for guidance only until they are confirmed.)	Late Submission Penalties (tick if appropriate)		74%
	Up to 1 week late (40% Max)		
	Over 1 week late (0%)		



ELECTRICAL PRINCIPLES THEORY

Fourier series and Fourier transform

Shaun Farthing

Lecturer – Dr Rabya Kadhim Module Code – M2X0003

Contents

1. Definitions.....	2
2. Square wave - sum of odd harmonics	3
3. Fourier trigonometric series representation	4
4. Verification of the spectral analysis.....	6
5. Conclusion	11
References.....	13

1. Definitions

Fourier series is expressed as,

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx] \quad \checkmark$$

Fourier series is an infinite series of harmonic sinusoidal components used to represent a periodic function (Collins English Dictionary, 2010). A Fourier series is a way to represent complex waves as a series of simple sine waves. The series breaks down the wave into a sum of sines and cosines. This means that the elements of a wave can be isolated from each other (Techopedia, 2018). In the equation above, a_0 refers to the fundamental frequency. a_n and b_n are harmonic components of the fundamental frequency, odd harmonics are odd multiples of the fundamental frequency and the even harmonics are even multiples of the fundamental frequency. (Kulkarni, 2002)
✓

Fourier transform decomposes any function into a sum of sinusoidal basis functions. Each of these basis functions is a complex exponential of a different frequency. The Fourier transform gives us a unique way of viewing any function as the sum of simple sinusoids. The Fourier series showed us how to re-write any periodic function into a sum of sinusoids. The Fourier transform is the extension of this idea to non-periodic functions (The Fourier Transform, 2010) (Elster, 2018)
✓

The Fourier transform is one of the most important mathematical tools in a wide variety of fields in science and engineering, It can be viewed as the transformation of a signal in one domain, (typically time or space) into another domain, the frequency domain. Applications of Fourier transforms, often called Fourier analysis or harmonic analysis, provide useful decompositions of complicated sums and integrals, and often reveal hidden structure in data. Fourier analysis lies at the base of many theories in science and plays a fundamental role in practical engineering design (Goodman, 1995) (Herman, 2016)
✓

2. Square wave - sum of odd harmonics

A square waveform is mathematically equivalent to the sum of a series of sine waves at that same frequency, plus an infinite series of odd-multiple frequency sine waves at diminishing amplitudes. (FourierSeries5ET, 2018) (Weissstein, 2019)

Image 1 below shows the sum of 3 odd harmonics being generated, it's still clear to see the ripples of the sine waves.

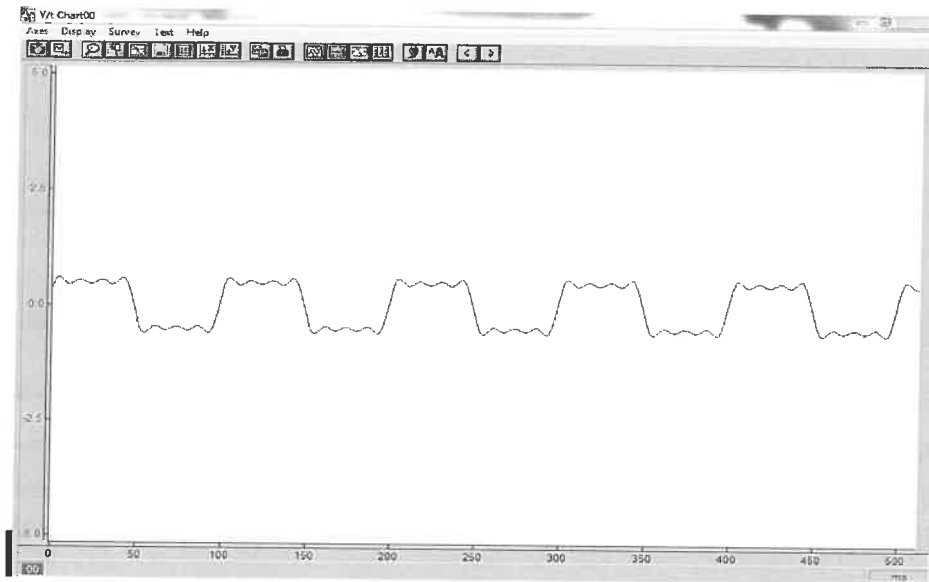
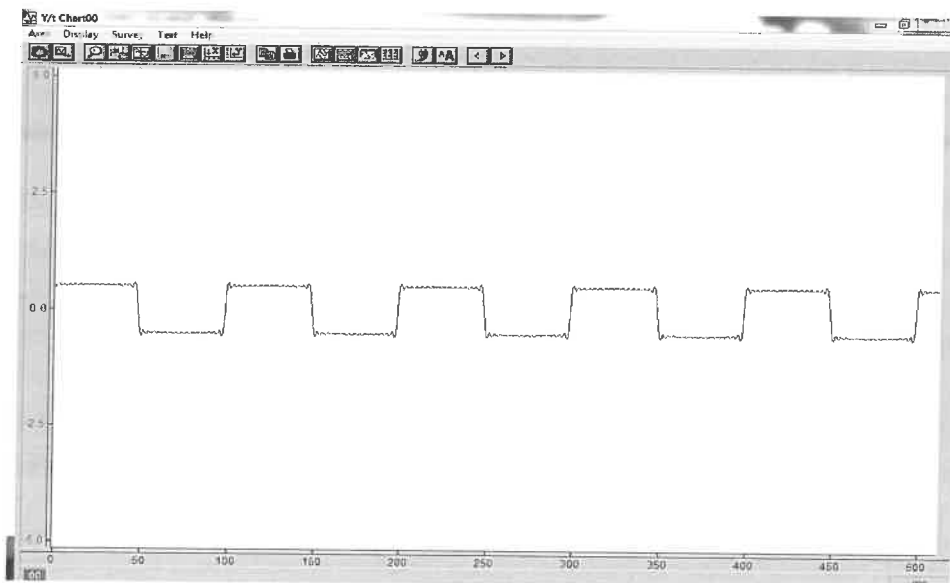


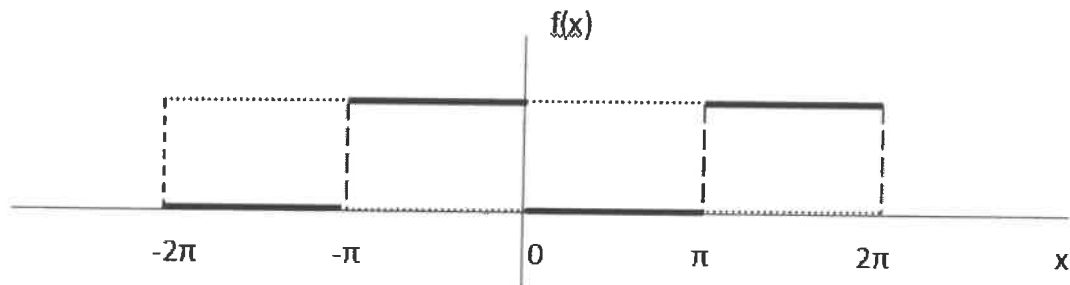
Image 2 below, shows 15 odd harmonics being generated, the sine waves generated appear to be squarer in appearance.



The more odd harmonics generated/ plotted the squarer the waveform will appear.

3. Fourier trigonometric series representation

Graph of $f(x)$ in the interval of -2π to $+2\pi$ below,



Where $f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi \end{cases}$

$$f(x) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

a_0

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^0 f(x) dx + \frac{1}{2\pi} \int_0^{\pi} f(x) dx$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^0 1 dx + \frac{1}{2\pi} \int_0^{\pi} 0 dx$$

$$a_0 = \frac{1}{2\pi} [x]_{-\pi}^0$$

$$a_0 = \frac{1}{2\pi} [\pi]$$

$$a_0 = \frac{1}{2}$$

an

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^0 1 \cos nx \, dx + \frac{1}{\pi} \int_0^{\pi} 0 \cos nx \, dx$$

$$a_n = \frac{1}{\pi} \left(\left[\frac{\sin nx}{n} \right]_{-\pi}^0 \right)$$

$$a_n = \frac{1}{n\pi} ([\sin nx]_{-\pi}^0)$$

$$a_n = \frac{1}{n\pi} [\sin n0 - \sin(-n\pi)]$$

$$a_n = \frac{1}{n\pi} [0 + \sin(n\pi)]$$

$$a_n = \frac{1}{n\pi} [0 + 0]$$

$$a_n = 0$$

bn

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^0 f(x) \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^0 1 \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} 0 \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \left[\left[-\frac{\cos nx}{n} \right]_{-\pi}^0 \right]$$

$$b_n = \frac{-1}{n\pi} [\cos nx]_{-\pi}^0$$

$$b_n = \frac{-1}{n\pi} [\cos 0 + \cos \pi]$$

$$b_n = \frac{-1}{n\pi} [1 + 1]$$

$$b_n = \frac{-2}{n\pi} \text{ when } n \text{ is odd}$$

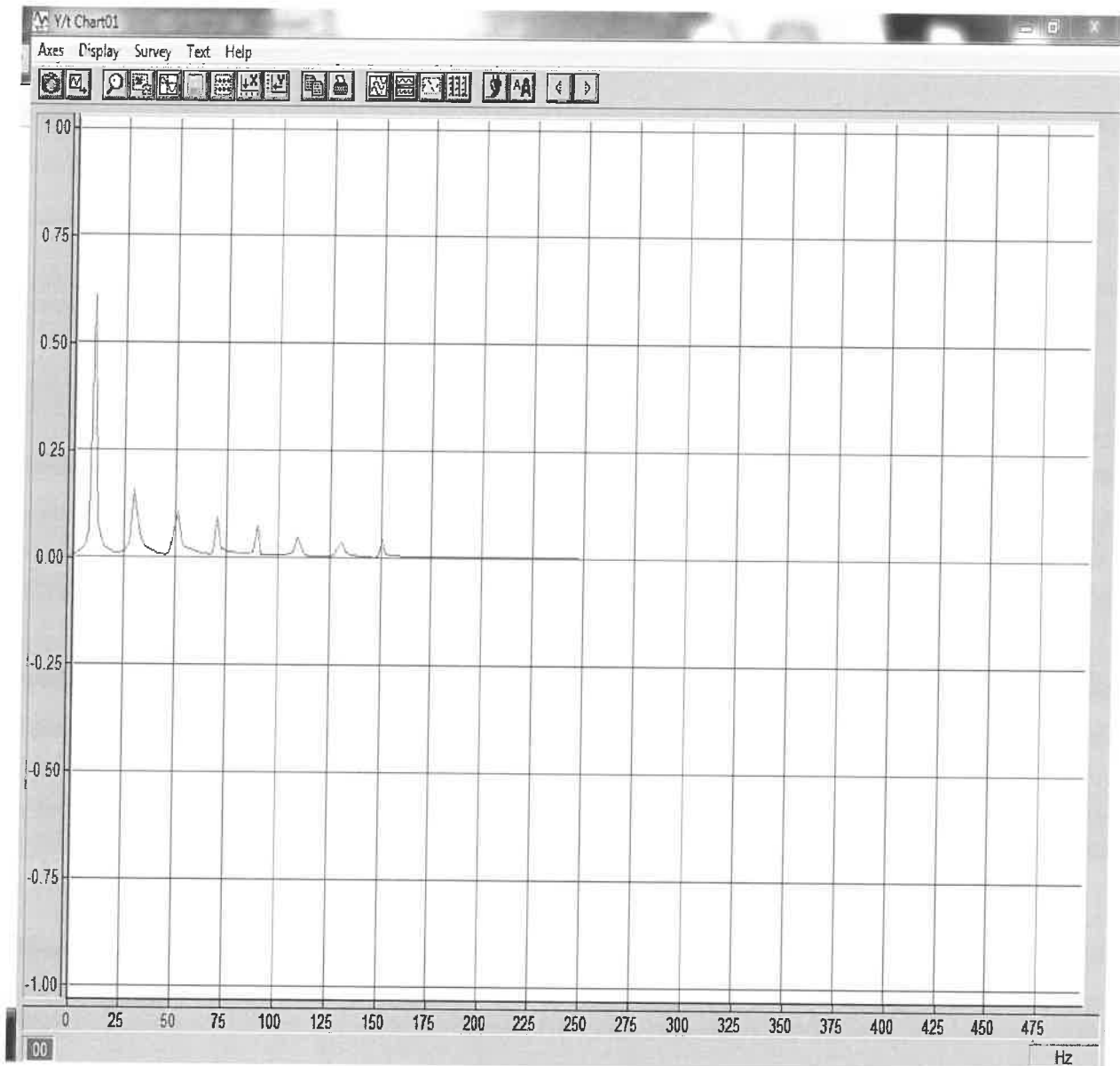
$$b_n = 0 \text{ when } n \text{ is even}$$

Therefore the fourier series representation for the first 7 harmonics is,

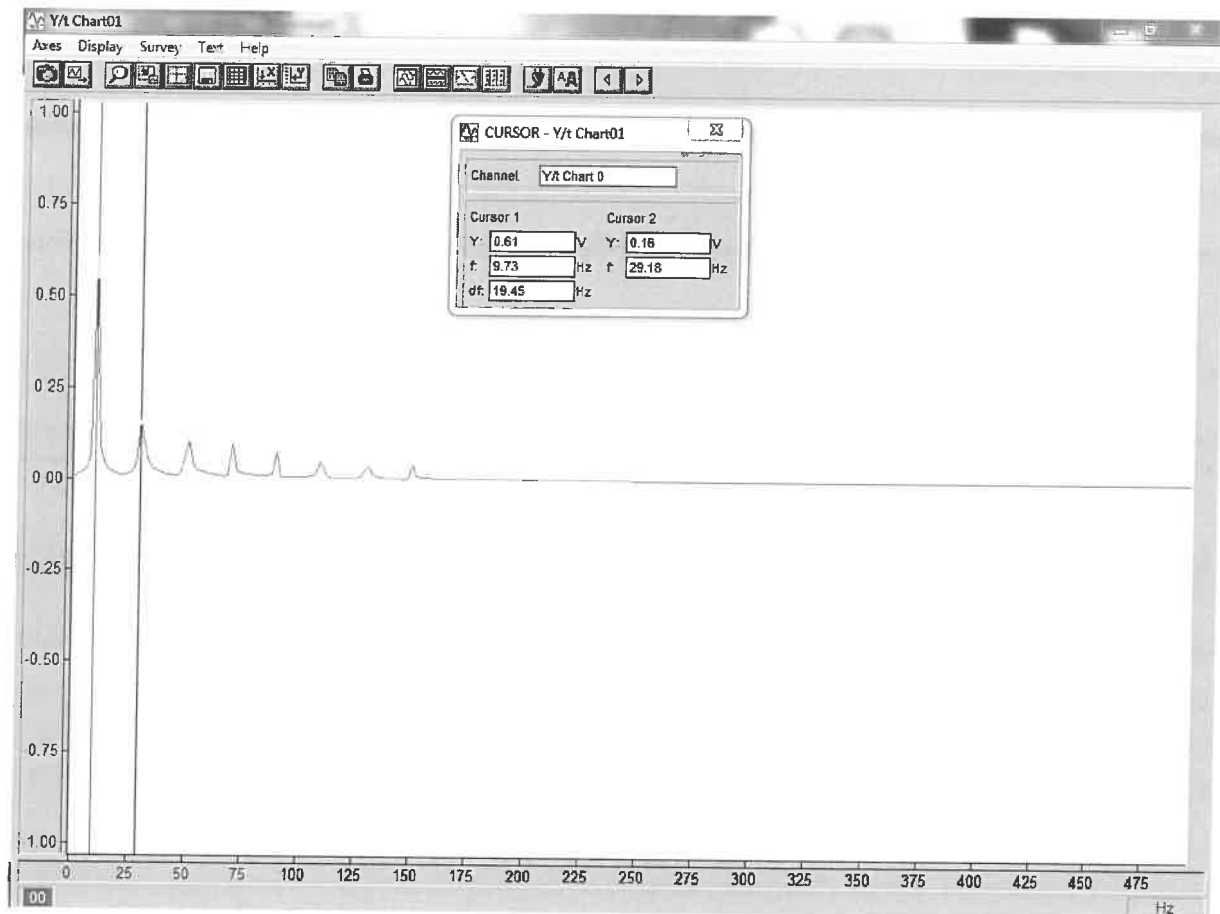
$$f(x) = \frac{1}{2} + \frac{-2}{\pi} \sin x + 0 \sin 2x + \frac{-2}{3\pi} \sin 3x + 0 \sin 4x + \frac{-2}{5\pi} \sin 5x + 0 \sin 6x + \frac{-2}{7\pi} \sin 7x$$

4. Verification of the spectral analysis

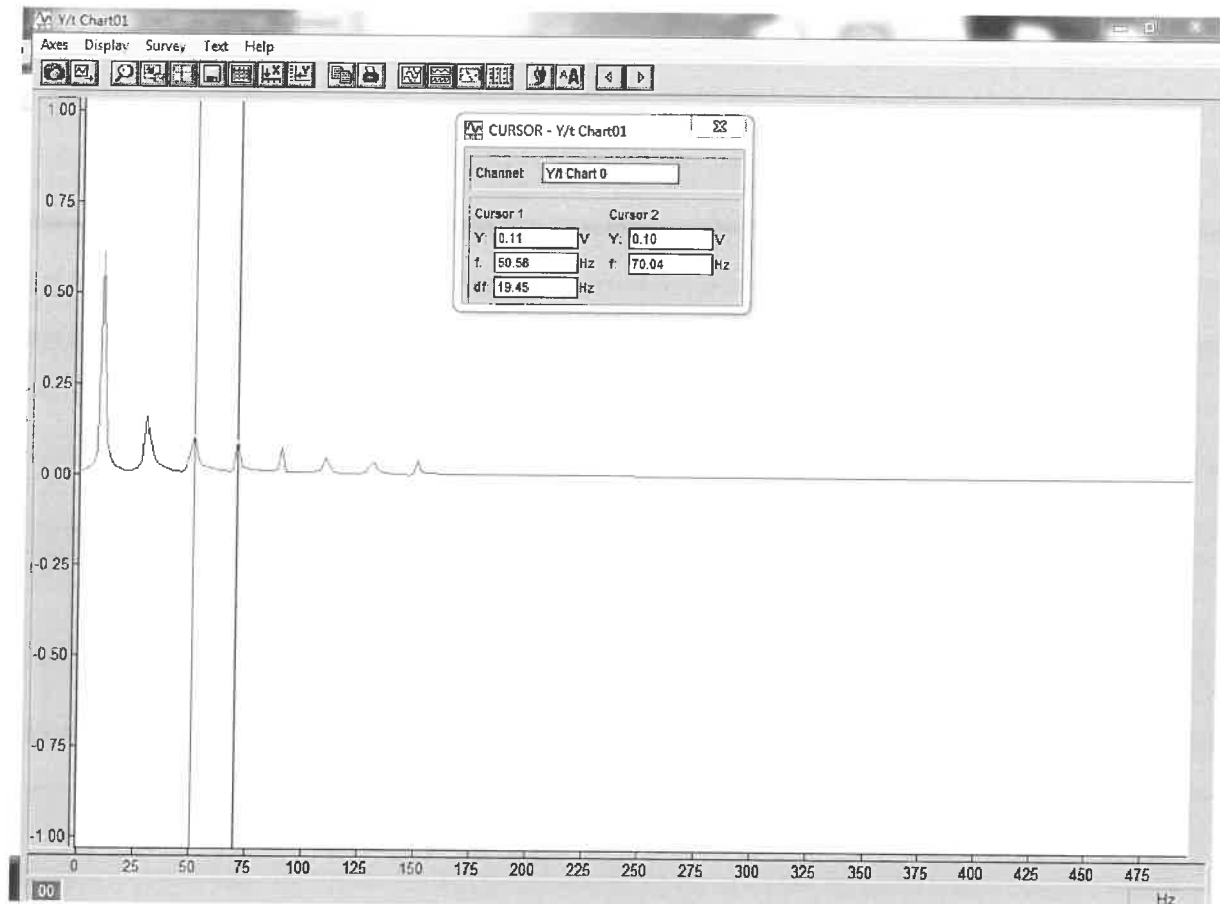
Using the results from task 3, I have used Fast Fourier transform to find the transform of a square wave. The results are plotted below.



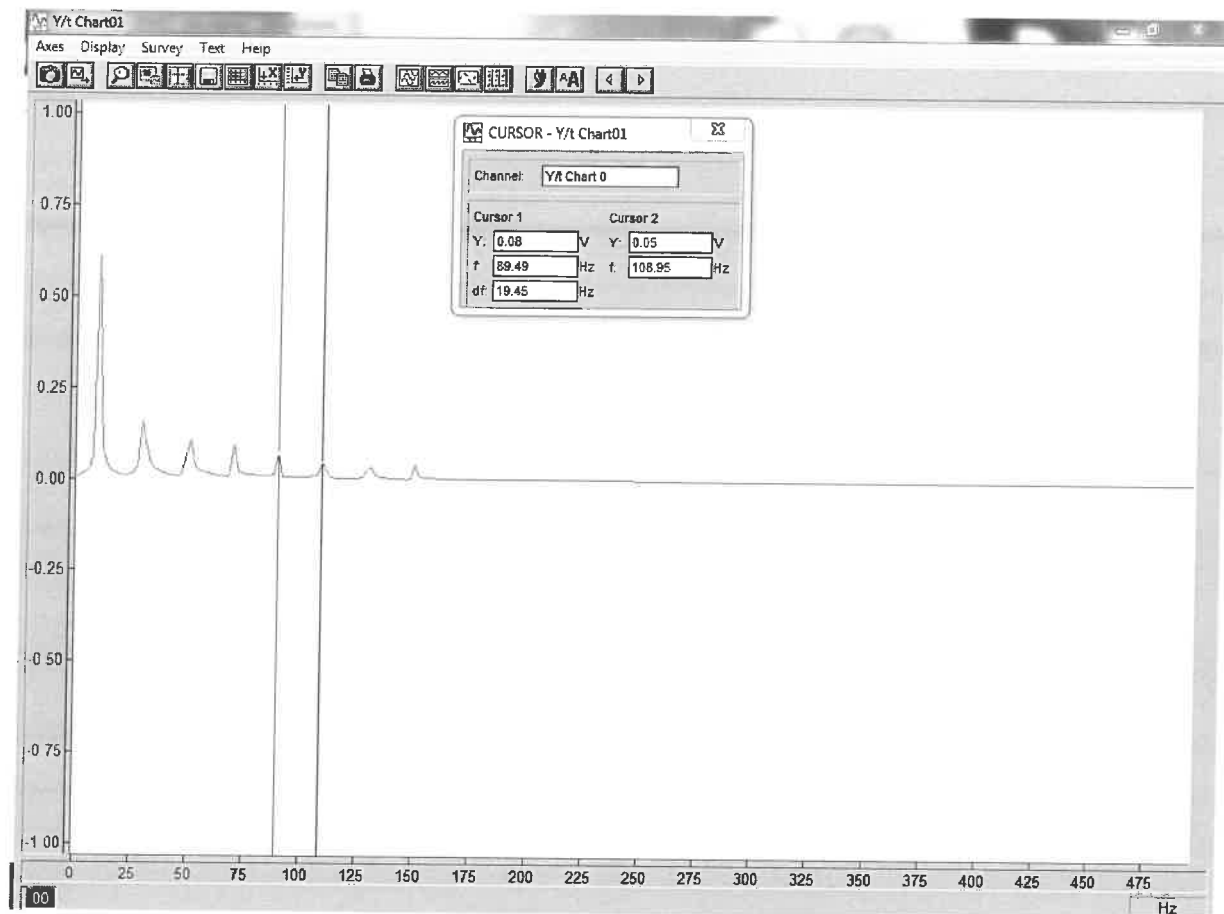
I have then used the cursor function in the software to compare each amplitude as plotted using FFT to the results calculated in task 3. The graphs are below,



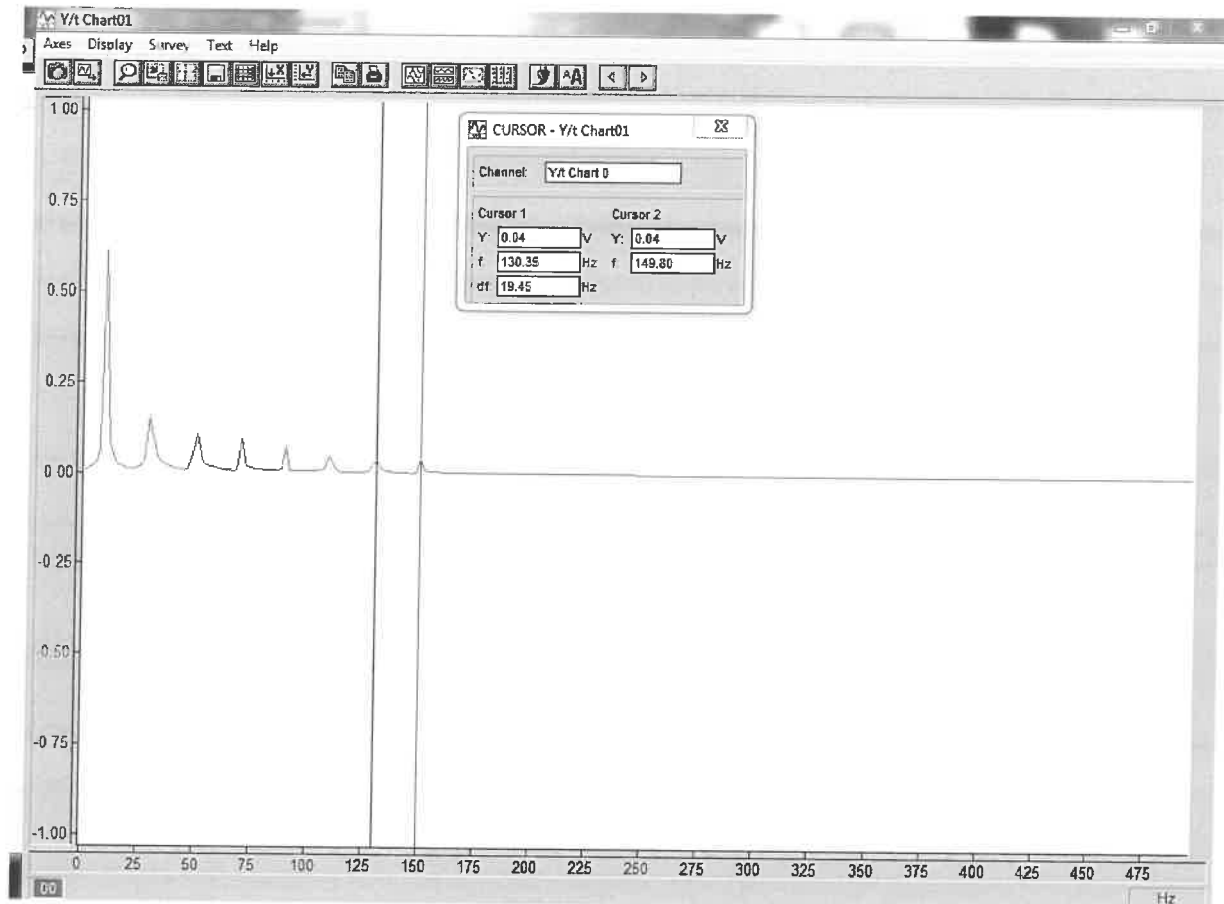
Cursor 1 shows the fundamental frequency, the plotted results show a frequency of 9.73Hz, (I had input 10Hz) and voltage of 0.61v. The calculated results from task 3 were an amplitude of 0.63v. Cursor 2 shows the first odd harmonic at 29.18Hz, (I had input 30Hz) and an amplitude of 0.16v. The calculated results from task 3 was an amplitude of 0.21v.



In the image above, cursor 1 shows the second odd harmonic at 50.58Hz, (I had input 50Hz) and an amplitude of 0.11v. The calculated results from task 3 was an amplitude of 0.12v. Cursor 2 shows the third odd harmonic at 70.04Hz, (I had input 70) and an amplitude of 0.10v. The calculated results from task 3 was an amplitude of 0.09v.



In the image above, cursor 1 shows the fourth odd harmonic at 89.49Hz, (I had input 90Hz) and an amplitude of 0.08v. The calculated results from task 3 was an amplitude of 0.07v. Cursor 2 shows the fifth odd harmonic at 108.95Hz, (I had input 110Hz) and an amplitude of 0.05v. The calculated results from task 3 was an amplitude of 0.057v.



In the image above, cursor 1 shows the sixth odd harmonic at 130.35Hz, (I had input 130Hz) and an amplitude of 0.04v. The calculated results from task 3 was an amplitude of 0.048v. Cursor 2 shows the seventh odd harmonic at 149.8Hz, (I had input 150Hz) and an amplitude of 0.04v. The calculated results from task 3 was an amplitude of 0.042v.

Although, the task only asks to plot results of the first 7 harmonics (odd and even) I have decided to display the first 7 odd harmonics to show the diminishing amplitude more clearly.

5. Conclusion

In conclusion, I have found that using FFT enables us to break down complex waveforms so that we can see what components make up that waveform. As an example, I have plotted the fundamental frequency and the first harmonic as a voltage with respect to time and the same signal using the FFT function.

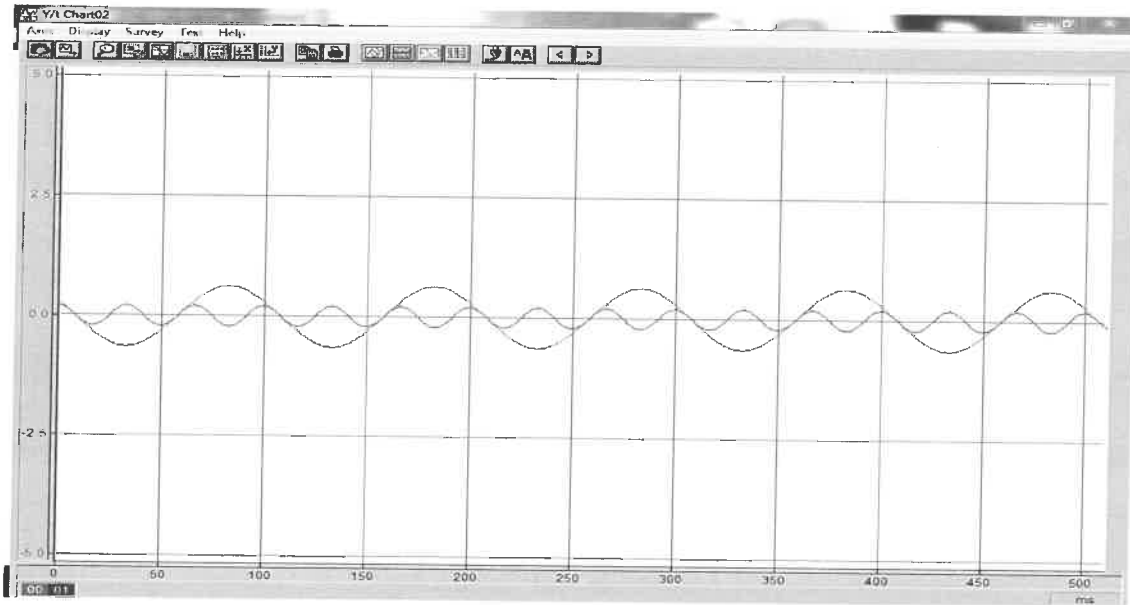
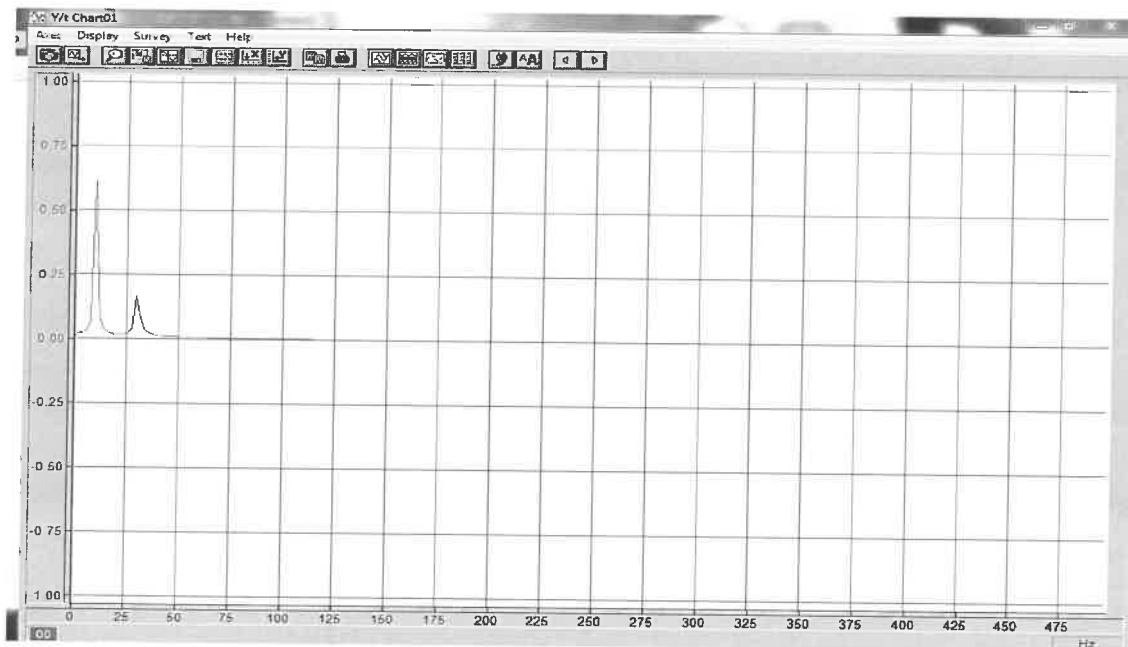


Image above shows the signals in the time domain.



The image above shows the signals in the frequency domain. Using FFT gives a clearer picture of the properties of the waveform.

The results that I have gained from both the calculation and the simulation are very closely matched.

Fundamental frequency – Plotted amplitude 0.61v, calculated amplitude 0.630v.

1st harmonic – Plotted amplitude 0.16v, calculated amplitude 0.210v.

3rd harmonic – Plotted amplitude 0.11v, calculated amplitude 0.127v.

5th harmonic – Plotted amplitude 0.10v, calculated amplitude 0.090v.

7th harmonic – Plotted amplitude 0.08v, calculated amplitude 0.070v.

Overall, I feel that the plotted results using the software confirms the results gained through calculation.

I feel that the simulation could have been improved if I was able to generate more sine waves from the one generation source. This would have enabled me to create a squarer wave form using sine waves.

Generally, I feel that the both the simulation and calculated results coincide and are a true reflection of the waveforms analysed.



References

- Collins English Dictionary. (2010). */dictionary/english/fourier-series*. Retrieved from www.collinsdictionary.com:
<https://www.collinsdictionary.com/dictionary/english/fourier-series>
- Elster, A. (2018). *Fourier transform ft*. Retrieved from MRI Questions:
<http://mriquestions.com/fourier-transform-ft.html>
- FourierSeries5ET*. (2018). Retrieved from stewartcalculus.com:
<https://www.stewartcalculus.com/data/CALCULUS%20Early%20Transcendentals/upfiles/FourierSeries5ET.pdf>
- Goodman, R. M. (1995). Fourier Transforms. In R. M. Goodman, *Fourier Transforms An Introduction for Engineers* (p. 361). Springer US.
- Herman, R. L. (2016). An Introduction to Fourier analysis. In R. L. Herman, *An Introduction to Fourier analysis* (p. 402). CRC Press.
- Kulkarni, S. R. (2002). *Frequency domain and fourier transforms*. Retrieved from Princeton:
https://www.princeton.edu/~cuff/ele201/kulkarni_text/frequency.pdf
- Techopedia. (2018). *definition/14381/fourier-series*. Retrieved from www.techopedia.com: <https://www.techopedia.com/definition/14381/fourier-series>
- The Fourier Transform. (2010). *introduction*. Retrieved from <http://www.thefouriertransform.com>:
<http://www.thefouriertransform.com/#introduction>
- Weissstein, E. W. (2019, January 9). *Fourier series*. Retrieved from Mathworld:
<http://mathworld.wolfram.com/FourierSeries.html>